Inverse Laplace transformation then yields $tI(\sqrt{t}) = t - 1 + e^{-t}$, from which we conclude that $I(a) = 1 - a^{-2}(1 - \exp -a^2)$. As in Solution I, this remains valid for $a < 0$.

Solved also by S. Amghibech (Canada), D. Beckwith, P. Bracken (Canada), M. Carone (Canada), J.-F. Chamayou (France), R. Chapman (U. K.), W. Chu (Italy), C. Curtis, K. Dale (Norway), N. Eklund, A. Eydelzon (Israel), P. Fitzsimmons, O. Furdui, B. D. Ganapol, M. L. Glasser, J. Grivaux (France), D. Henderson, C. Hill, B. H. Margolius, K. McInturff, M. A. Prasad (India), J. Rouse, A. Stadler (Switzerland), R. Stong, D. B. Tyler, E. Wepsic, L. Zhou, GCHQ Problem Solving Group, NSA Problems Group, and the proposer.

Powered Terms in a Series

11001 [2003, 240, correction 543]. *Proposed by Rick Mabry, LSU-Shreveport, Shreveport, LA.* Let $\langle a_n \rangle$ be a sequence of real numbers.
(a) Given that $\sum_1^\infty a_n$ converges and that p is an odd integer greater than 1, must $\sum_1^\infty a_n^p$ converge?
(b) Again given that $\sum a_n$ converges, must there exist a positive integer P such that $\sum a_n^p$ converges whenever p is an odd integer greater than P?
(c) Given that all a_n are positive and that $\sum (-1)^n a_n$ converges, must there be a positive integer P such that $\sum (-1)^n a_n^p$ converges whenever p is an odd integer greater than P?

Solution by Jody Lockhart, U.S. Naval Academy, Annapolis, MD. We show that the answer to **(c)** is no by giving a convergent series $\sum (-1)^n a_n$ with all a_n positive such that $\sum (-1)^n a_n^p$ diverges for all odd integers $p > 1$. The same series, where a_n includes the $(-1)^n$, shows that the answers to **(a)** and **(b)** are also no.

For $k \geq 0$, define $a_{4k+1} = a_{4k+3} = 1/\ln(k+2)$, $a_{4k+2} = 2/\ln(k+2)$, and $a_{4k+4} = \frac{1}{k+1} - \frac{1}{k+2}$. If $n = 4N + j$ with $0 \leq j \leq 3$, then the partial sum s_n is given by

$$s_n = \sum_{k=1}^{n} (-1)^k a_k = 1 - \frac{1}{N+1} + \frac{e_j}{\ln(N+2)},$$

where $e_j \in \{-1, 0, 1\}$. Thus, s_n approaches 1 as n grows, and $\sum (-1)^n a_n$ converges.

Let p be odd and greater than 1. For the modified series $\sum (-1)^n a_n^p$, the partial sums are given by

$$s_{4N} = \sum_{k=1}^{4N} (-1)^k a_k^p = \sum_{k=1}^{N} \frac{1}{(k^2+k)^p} + \sum_{k=2}^{N+1} \frac{2^p - 2}{(\ln k)^p}.$$

The first sum converges; the second diverges. Thus the partial sums $\sum_{k=1}^{4N} (-1)^k a_k^p$ must diverge. Hence $\sum_1^\infty (-1)^n a_n^p$ also diverges.

Also solving the corrected problem were R. Bagby, D. Borwein (Canada), P. Budney, T. K. Callahan, K. Dutch, J. P. Grivaux (France), J. Hagood, G. Heuer, J. Lindsey, O. P. Lossers (Netherlands), A. Stenger, R. Stong, A. Witkowski, BSI Problems Group (Germany), Szeged Problem Group Fejéntaláltuka (Hungary), GCHQ Problem Solving Group (U. K.), University of Louisiana–Lafayette Math Club, and the proposer.

A Problem That Bears Repeating

11002 [2003, 240]. *Proposed by Y.-F. S. Pétermann, Université de Genève, Geneva, Switzerland.* Pooh Bear has $2N + 1$ honey pots. No matter which one of them he sets aside, he can split the remaining $2N$ pots into two sets of the same total weight, each consisting of N pots. Must all $2N + 1$ pots weigh the same?

Solution I by C. T. Stretch, University of Ulster at Coleraine, County Londonderry, Northern Ireland. Yes, they must weigh the same.